

Relational Algebra

Chapter 4

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Mohammad Sadoghi

Exploratory Systems Lab

Department of Computer Science

UC DAVIS
UNIVERSITY OF CALIFORNIA



Relational Query Languages

- ❖ Query languages: Allow manipulation and **retrieval of data** from a database.
- ❖ Relational model supports simple, powerful QLs:
 - Strong formal foundation based on logic.
 - Allows for much optimization.
- ❖ Query Languages **!=** programming languages!
 - QLs not expected to be “Turing complete”.
 - QLs not intended to be used for complex calculations.
 - QLs support easy, efficient access to large data sets.

Formal Relational Query Languages

- ❖ Two mathematical Query Languages form the basis for “real” languages (e.g. SQL), and for implementation:
 - Relational Algebra: More **operational**, very useful for representing execution plans.
 - Relational Calculus: Lets users describe what they want, rather than how to compute it. (**Non-operational, declarative.**)

Preliminaries

- ❖ A query is applied to *relation instances*, and the result of a query is also a relation instance.
 - *Schemas of input* relations for a query are **fixed** (but query will run regardless of instance!)
 - The **schema for the result** of a given query is also **fixed!** Determined by definition of query language constructs.

Example Instances

“Sailors” and “Reserves”
relations for our examples.

S1

<u>sid</u>	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S2

<u>sid</u>	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

R1

<u>sid</u>	<u>bid</u>	<u>day</u>
22	101	10/10/96
58	103	11/12/96

Relational Algebra

❖ Basic operations:

- Selection (σ) Selects a subset of rows from relation.
- Projection (Π) Deletes unwanted columns from relation.
- Cross-product (\times) Allows us to combine two relations.
- Set-difference ($-$) Tuples in reln. 1, but not in reln. 2.
- Union (\cup) Tuples in reln. 1 and in reln. 2.

❖ Additional operations:

- Intersection, join, division, renaming: Not essential, but (very!) useful.

❖ Since each operation returns a relation, **operations can be composed!** (Algebra is “closed”.)

Projection

- ❖ Deletes attributes that are not in *projection list*.
- ❖ *Schema* of result contains exactly the fields in the projection list, with the same names that they had in the (only) input relation.
- ❖ Projection operator has to eliminate *duplicates!* (Why??)
 - Note: real systems typically don't do duplicate elimination unless the user explicitly asks for it. (Why not?)

sname	rating
yuppy	9
lubber	8
guppy	5
rusty	10

$\pi_{sname, rating}(S2)$

age
35.0
55.5

$\pi_{age}(S2)$

Selection

- ❖ Selects rows that satisfy *selection condition*.
- ❖ No duplicates in result! (Why?)
- ❖ *Schema* of result identical to schema of (only) input relation.
- ❖ *Result* relation can be the *input* for another relational algebra operation! (*Operator composition*.)

S2

<u>sid</u>	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

sid	sname	rating	age
28	yuppy	9	35.0
58	rusty	10	35.0

$$\sigma_{rating > 8}(S2)$$

sname	rating
yuppy	9
rusty	10

$$\pi_{sname, rating}(\sigma_{rating > 8}(S2))$$

Union, Intersection, Set-Difference

❖ All of these operations take two input relations, which must be union-compatible:

- Same number of fields.
- 'Corresponding' fields have the same type.

❖ What is the *schema* of result?

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S1

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

S2

$S1 \cap S2$

sid	sname	rating	age
31	lubber	8	55.5
58	rusty	10	35.0

$S1 - S2$

sid	sname	rating	age
22	dustin	7	45.0

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0
44	guppy	5	35.0
28	yuppy	9	35.0

$S1 \cup S2$

Cross-Product

<u>sid</u>	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S1

<u>sid</u>	<u>bid</u>	<u>day</u>
22	101	10/10/96
58	103	11/12/96

R1

- ❖ Each row of *S1* is paired with each row of *R1*.
- ❖ *Result schema* has one field per field of *S1* and *R1*, with field names 'inherited' if possible.
 - *Conflict*: Both *S1* and *R1* have a field called *sid*.

(sid)	sname	rating	age	(sid)	bid	day
22	dustin	7	45.0	22	101	10/10/96
22	dustin	7	45.0	58	103	11/12/96
31	lubber	8	55.5	22	101	10/10/96
31	lubber	8	55.5	58	103	11/12/96
58	rusty	10	35.0	22	101	10/10/96
58	rusty	10	35.0	58	103	11/12/96

- Renaming operator: $\rho (C(1 \rightarrow sid1, 5 \rightarrow sid2), S1 \times R1)$

Joins

<u>sid</u>	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S1

<u>sid</u>	<u>bid</u>	<u>day</u>
22	101	10/10/96
58	103	11/12/96

R1

❖ Condition Join: $R \bowtie_c S = \sigma_c(R \times S)$

(sid)	sname	rating	age	(sid)	bid	day
22	dustin	7	45.0	58	103	11/12/96
31	lubber	8	55.5	58	103	11/12/96

$$R \bowtie_{S1.sid < R1.sid} S$$

- ❖ *Result schema* same as that of cross-product.
- ❖ Fewer tuples than cross-product, might be able to compute more efficiently
- ❖ Sometimes called a *theta-join*.

Joins

<u>sid</u>	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S1

<u>sid</u>	<u>bid</u>	<u>day</u>
22	101	10/10/96
58	103	11/12/96

R1

- ❖ Equi-Join: A special case of condition join where the condition c contains only *equalities*.

sid	sname	rating	age	bid	day
22	dustin	7	45.0	101	10/10/96
58	rusty	10	35.0	103	11/12/96

$$R \bowtie_{sid} S$$

- ❖ Result schema similar to cross-product, but only one copy of fields for which equality is specified.
- ❖ Natural Join: Equijoin on *all* common fields.

Division

- ❖ Not supported as a primitive operator, but useful for expressing queries like:
 - Find sailors who have reserved all boats.*
- ❖ Let A have 2 fields, x and y ; B have only field y :
 - $A/B = \{ \langle x \rangle \mid \exists \langle x, y \rangle \in A \ \forall \langle y \rangle \in B \}$
 - i.e., **A/B contains all x tuples (sailors) such that for every y tuple (boat) in B , there is an xy tuple in A .**
 - Or: If the set of y values (boats) associated with an x value (sailor) in A contains all y values in B , the x value is in A/B .
- ❖ In general, x and y can be any lists of fields; y is the list of fields in B , and $x \cup y$ is the list of fields of A .

Examples of Division A/B

sno	pno
s1	p1
s1	p2
s1	p3
s1	p4
s2	p1
s2	p2
s3	p2
s4	p2
s4	p4

A

pno
p2

B1

sno
s1
s2
s3
s4

A/B1

pno
p2
p4

B2

sno
s1
s4

A/B2

pno
p1
p2
p4

B3

sno
s1

A/B3

Expressing A/B Using Basic Operators

- ❖ Division is not essential op; just a useful shorthand.
 - (Also true of joins, but joins are so common that systems implement joins specially.)
- ❖ *Idea:* For A/B , compute all x values that are not 'disqualified' by some y value in B .
 - x value is *disqualified* if by attaching y value from B , we obtain an xy tuple that is not in A .

Disqualified x values: $\pi_x((\pi_x(A) \times B) - A)$

A/B : $\pi_x(A) -$ all disqualified tuples

<u>sid</u>	<u>sname</u>	<u>rating</u>	<u>age</u>
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S1

<u>sid</u>	<u>bid</u>	<u>day</u>
22	101	10/10/96
58	103	11/12/96

R1

Find names of sailors who've reserved boat #103

❖ **Solution 1:** $\pi_{sname}((\sigma_{bid=103}Reserves) \bowtie Sailors)$

❖ **Solution 2:** $\rho(Temp1, \sigma_{bid=103}Reserves)$

$\rho(Temp2, Temp1 \bowtie Sailors)$

$\pi_{sname}(Temp2)$

❖ **Solution 3:** $\pi_{sname}(\sigma_{bid=103}(Reserves \bowtie Sailors))$

<u>sid</u>	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S1

<u>sid</u>	<u>bid</u>	<u>day</u>
22	101	10/10/96
58	103	11/12/96

R1

Find names of sailors who've reserved a red boat

- ❖ Information about boat color only available in Boats; so need an extra join:

$$\pi_{sname}((\sigma_{color='red'}Boats) \bowtie Reserves \bowtie Sailors)$$

- ❖ A more efficient solution:

$$\pi_{sname}(\pi_{sid}((\pi_{bid}\sigma_{color='red'}Boats) \bowtie Reserves) \bowtie Sailors))$$

A query optimizer can find this, given the first solution!

<u>sid</u>	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S1

<u>sid</u>	<u>bid</u>	<u>day</u>
22	101	10/10/96
58	103	11/12/96

R1

Find sailors who've reserved a red or a green boat

- ❖ Can identify all red or green boats, then find sailors who've reserved one of these boats:

$$\rho(\text{Tempboats}, (\sigma_{color='red' \vee color='green'} \text{Boats}))$$

$$\pi_{sname}(\text{Tempboats} \bowtie \text{Reserves} \bowtie \text{Sailors})$$

- ❖ Can also define Tempboats using union! (How?)
- ❖ What happens if \vee is replaced by \wedge in this query?

<u>sid</u>	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

<u>sid</u>	<u>bid</u>	<u>day</u>
22	101	10/10/96
58	103	11/12/96

R1

S1

Find sailors who've reserved a red and a green boat

- ❖ Previous approach won't work! Must identify sailors who've reserved red boats, sailors who've reserved green boats, then find the intersection (note that *sid* is a key for Sailors):

$$\rho(\text{Tempred}, \pi_{sid}((\sigma_{color='red'}\text{Boats}) \bowtie \text{Reserves}))$$

$$\rho(\text{Tempgreen}, \pi_{sid}((\sigma_{color='green'}\text{Boats}) \bowtie \text{Reserves}))$$

$$\pi_{sname}((\text{Tempred} \cap \text{Tempgreen}) \bowtie \text{Sailors})$$

<u>sid</u>	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S1

<u>sid</u>	<u>bid</u>	<u>day</u>
22	101	10/10/96
58	103	11/12/96

R1

Find the names of sailors who've reserved all boats

- ❖ Uses division; schemas of the input relations to / must be carefully chosen:

$$\rho(\text{TempSids}, (\pi_{sid,bid} \text{Reserves}) / (\pi_{bid} \text{Boats}))$$

$$\pi_{sname}(\text{TempSids} \bowtie \text{Sailors})$$

- ❖ To find sailors who've reserved all 'Interlake' boats:

$$\dots / \pi_{bid}(\sigma_{bname='Interlake'} \text{Boats})$$

Summary

- ❖ The relational model has rigorously defined query languages that are simple and powerful.
- ❖ Relational algebra is more operational; useful as internal representation for query evaluation plans.
- ❖ Several ways of expressing a given query; a query optimizer should choose the most efficient version.